Singaporean Journal of Scientific Research

Singaporean Journal of Scientific Research(SJSR)
Journal of Technology and Engineering System(JTES)
Vol.8.No.3 2016 Pp.227-238
available at :www.iaaet.org/sjsr
Paper Received : 28-08-2016
Paper Accepted: 19-09-2016
Paper Reviewed by: 1.Prof. Cheng Yu 2. Dr.M. Akshay Kumar
Editor : Dr. Chu Lio

FIXED POINT THEOREMS FOR GENERALIZED CIRIC-BERINDE TYPE CONTRACTIVE MULTIVALUED MAPPINGS

U. Karuppiah[1] and M. Gunaseelan[2]

1. Department of Mathematics,
St.Joseph's College(Autonomous),
Tiruchirappalli-620 002.
E-mail:u.karuppiah@gmail.com

2. Department of Mathematics,
Sri Sankara Arts and Science College,
Enathur, Kanchipuram-631 561.
E-mail:mathsguna@yahoo.com

Abstract

The purpose of this paper is to establish a generalized Ćirić-Berinde type contractive for multivalued mappings and to give some application in this support of established results.The result presented in this paper extent and generalize some previous work given in existing literature of [1].

Key words: α_*-admissible multivalued mapping;α-admissible multivalued mapping;Fixed ponit.

1 Introduction and Preliminaries

In 2012,Samet et al.[2] introduced the notions of α-ψ-contractive mappings and α-admissible mappings in metric spaces and obtained corresponding

fixed pint results,which are generalizations of ordered fixed point results(see[2]). Since then,by using their idea,some authors investigated fixed point results in the field.Asl et al.[3] extended some of results in [2] to multivalued mappings by introducing the notions of α_*-ψ-contractive mappings and α_*-admissible mappings.
Recently,salimi et al.[4] modified the notions of α-ψ-contractive mapping and α-admissible by introducing another function η.And then,they gave generalizations of the results of samet et al.[2]and Karapinar and samet[5].Hussain et al.[6]extended these modified notions to multivalued mappings.That is,they introduced the notion of α-η-contractive multifunctions and gave fixed point results for these multifunctions.
Recently,Ali et al.[7] generalized and extended the notion of α-ψ-contractive mappings by introducing the notion of (α,ψ,ξ)-contracive multivalued mappings and obtained fixed point theorems for these mappings in complete metric spaces.
Very recently,Seong-Hoon Cho.[6] introduce the notion of Ćirić-Berinde type contractive multivalued mappings and to generalize and extend the notion ofα-η-contractive multifunctions and to establish fixed point theorems for Ćirić-Berinde type contractive multivalued mappings.
Let (X, d) be a metric space.We denote by $CB(X)$ the class of nonempty closed and bounded subsets of X and by $CL(X)$ the class of nonempty closed subsets of X.Let $H(.,.)$ be the generalized Hausdorff distance on $CL(X)$;that is,for all $A, B \in CL(X)$,

$$\mathrm{H}(A, B) = \begin{cases} \max\{\sup_{a\in A} d(a, B), \sup_{b\in B} d(b, A)\} & if the \quad maximum \quad exists \\ \infty & if \quad otherwise \end{cases} \tag{1}$$

where $d(a, B) = \inf\{d(a, b) : b \in B\}$ is the distance from point a to subset B.
For A,$B \in CL(X)$,let $D(A, B) = \sup_{x\in A} \inf_{y\in B}(x, y)$.
Then,we have $D(A, B) \leq H(A, B)$ for all $A, B \in CL(X)$.
We denote by Ξ the class of all functions $\xi : [0, \infty) \to [0, \infty)$ such that
(1) ξ is continuous;
(2) ξ is nondecreasing on $[0, \infty)$;
(3) $\xi(t) = 0$ if and only if $t = 0$;
(4) ξ is subadditive.
Also,we denote by Ψ the family of all nondecreasing functions $\psi : [0, \infty) \to [0, \infty)$ such that $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for each $t > 0$,where ψ^n is the nth iterate

of ψ.
Note that if $\psi \in \Psi$,then $\psi(0) = 0$ and $0 < \psi(t) < t$ for all $t > 0$.
Let (X, d) be a metric space,and let $\alpha\colon X \times X \to [0, \infty)$ be a function .
We consider the following conditions:
(1) for any sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} x_n = x$,we have
$\alpha(x_n, x) \geq 1 \ \forall \ n \in \mathbb{N}$;
(2)for any sequence x_n in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and a cluster point x of x_n,we have $\lim_{n\to\infty} \inf \alpha(x_n, x) \geq 1$;
(3) for any sequence x_n in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and a cluster point x of x_n,there exists a subsequence $x_n(k)$ of x_n such that $\alpha(x_n(k), x) \geq 1 \ \forall k \in \mathbb{N}$.

Remark 1.1. *(1) implies (2) and (2) implies (3).*

Note that if (X, d) is a metric space and $\xi \in \Xi$,then $(X, \xi_\circ d)$ is a metric space.
Let (X, d) be a metric space,and let $T\colon X \to CL(X)$ be a multivalued mapping.Then,we say that
(1)T is called α_*-admissible[3] if $\alpha(x, y) \geq 1$ implies $\alpha_*(Tx, Ty) \geq 1$,
where $\alpha_*(Tx, Ty) = inf\{\alpha(a, b) : a \in Tx, b \in Ty\}$;
(2)T is called α-admissible[8]if,for each $x \in X$ and $y \in Tx$ with $\alpha(x, y) \geq 1$,we have $\alpha(y, z) \geq 1 \ \forall \ z \in Ty$.

Lemma 1.2. ([1]) *Let (X, d) be a metric space,and let $T\colon X \to CL(X)$ be a multivalued mappings.If T is α_*-admissible,then it is α-admissible.*

Lemma 1.3. ([1]) *Let (X, d) be a metric space,and let $\xi \in \Xi$ and $B \in CL(X)$.*
If $a \in X$ and $\xi(d(a, B)) < c$,then there exists $b \in B$ such that $\xi(d(a, b)) < c$.

Let (X, d) be a metric space.
A function $f : X \to [0, \infty)$ is called upper semicontinuous is for each $x \in X$ and $\{x_n\} \subset X$ with $\lim_{n\to\infty} x_n = x$,we have $\lim_{n\to\infty} f(x_n) \leq f(x)$.
A function $f\colon X \to [0, \infty)$ is called lower semicontinuous if,for each $x \in X$and $\{x_n\} \subset X$ with $\lim_{n\to\infty} x_n = x$,we have $f(x) \leq \lim_{n\to\infty} f(x_n)$.
For a multivalued map $T\colon X \to CL(X)$,let $f_T\colon X \to [0, \infty)$ be a function defined by $f_T(x) = d(x, Tx)$.

Theorem 1.4. ([1]) *Let (X, d) be a comxplete metric space,and let $\alpha\colon X \times X \to [0, \infty)$ be a function.suppose that a multivalued mappings $T\colon X \to$*

$CL(X)$is α-admissible. Assume that for all $x, y \in X$,$\alpha(x, y) \geq 1$ implies
$\xi(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx))$,
where $L \geq 0$,$\xi \in \Xi$,

$$M(x, y) = \max\{d(x, y), d(x, Tx), d(y, Ty), \frac{1}{2}\{d(x, Ty) + d(y, Tx)\}\} \quad (2)$$

and $\psi \in \Psi$is strictly increasing .
Also,suppose that the following are satisfied:
(1)there exists $x_0 \in X$and $x_1 \in Tx_0$such that $\alpha(x_0, x_1) \geq 1$;
(2)either T is continuous or f_T is lower semicontinuous.
Then T has a fixed point in X.

Theorem 1.5. ([1]) *Let (X, d) be a complete metric space,and let $\alpha\colon X \times X \to [0, \infty)$ be a function.Suppose that a multivalued mappings $T\colon X \to CL(X)$ is $\alpha(x, y) \geq 1$implies*
$\xi(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx))$,
where $L \geq 0$,$\xi \in \Xi$,

$$M(x, y) = \max\{d(x, y), d(x, Tx), d(y, Ty), \frac{1}{2}\{d(x, Ty) + d(y, Tx)\}\} \quad (3)$$

and $\psi \in \Psi$is strictly increasing and upper semicontinuous function.Also suppose that the following are satisfied :
(1) there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$;
(2)for a sequence x_n in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N} \cup \{0\}$ and a cluster point x of $\{x_n\}$,there exists a subsequence $\{x_n(k)\}$ of $\{x_n\}$ such that,for all $k \in \mathbb{N} \cup \{0\}$,
$\alpha(x_n(k), x) \geq 1$.
Then T has a fixed point in X.

In this paper,we introduce the notion of Ćirić-Berinde type contractive multivalued mappings and to generalize the above results.

2 Fixed Point Theorems

Theorem 2.1. *Let (X, d) be a comxplete metric space,and let $\alpha\colon X \times X \to [0, \infty)$ be a function.suppose that a multivalued mappings $T\colon X \to CL(X)$is α-admissible.*
Assume that for all $x, y \in X$,$\alpha(x, y) \geq 1$ implies

$$\xi(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx)), \quad (4)$$

where $L \geq 0$,$\xi \in \Xi$,

$$M(x,y) = \max\{d(x,y), \frac{d(x,Tx)d(y,Ty)}{d(x,y)}, \frac{1}{2}\{d(x,Ty)+d(y,Tx)\}\} \qquad (5)$$

and $\psi \in \Psi$*is strictly increasing .*
Also,suppose that the following are satisfied:
(1)there exists $x_0 \in X$*and* $x_1 \in Tx_0$*such that* $\alpha(x_0,x_1) \geq 1$;
(2)either T *is continuous or* f_T *is lower semicontinuous.*
Then T *has a fixed point in* X.

Proof. Let $x_0 \in X$ and $x_1 \in Tx_0$ be such that $\alpha(x_0,x_1) \geq 1$.
Let c be a real number with $\xi(d(x_0,x_1)) < \xi(c)$.
Let $x_0 \neq x_1$.
If $x_1 \in Tx_1$,then x_1 is a fixed point.Let $x_1 \notin Tx_1$.Then
$d(x_1,Tx_1) > 0$.
From (4) we obtain
$0 < \xi(d(x_1,Tx_1))$

$$\leq \xi(H(Tx_0,Tx_1))$$
$$\leq \psi(\xi(\max\{d(x_0,x_1), \frac{d(x_0,Tx_0)d(x_1,Tx_1)}{d(x_0,x_1)}, \frac{1}{2}\{d(x_0,Tx_1)+d(x_1,Tx_0)\}\})) + L\xi(d(x_1,Tx_0))$$
$$\leq \psi(\xi(\max\{d(x_0,x_1), \frac{d(x_0,x_1)d(x_1,Tx_1)}{d(x_0,x_1)}, \frac{1}{2}\{d(x_0,Tx_1)+d(x_1,x_1)\}\})) + Ld(x_1,x_1)$$
$$\leq \psi(\xi(\max\{d(x_0,x_1), d(x_1,Tx_1), \frac{1}{2}\{d(x_0,x_1)+d(x_1,Tx_1)\}\}))$$
$$\leq \psi(\xi(\max\{d(x_0,x_1), d(x_1,Tx_1)\}))$$

If $\max\{d(x_0,x_1), d(x_1,Tx_1)\} = d(x_1,Tx_1)$,then we have
$0 < \xi(d(x_1,Tx_1)) \leq \psi(\xi(d(x_1,Tx_1))) < \xi(d(x_1,Tx_1))$ which is a contradiction.
Thus,$\max\{d(x_0,x_1), d(x_1,Tx_1)\} = d(x_0,x_1)$,and hence we have
$0 < \xi(d(x_1,Tx_1)) \leq \psi(\xi(d(x_0,x_1))) < \psi(\xi(c))$.
Hence,there exists $x_2 \in Tx_1$ such that $\xi(d(x_1,x_2)) < \psi(\xi(c))$
Since,T is a α-admissible,from condition(1) and $x_2 \in Tx_1$,
we have $\alpha(x_1,x_2) \geq 1$.
If $x_2 \in Tx_2$,then x_2 is a fixed point.Let $x_2 \notin Tx_2$.
Then $d(x_2,Tx_2) > 0$,and so $\xi(d(x_2,Tx_2)) > 0$.
From (4) we obtain

$$
\begin{aligned}
0 &< \xi(d(x_2, Tx_2)) \\
&\leq \xi(H(Tx_1, Tx_2)) \\
&\leq \psi(\xi(\max\{d(x_1, x_2), \frac{d(x_1, Tx_1)d(x_2, Tx_2)}{d(x_1, x_2)}, \frac{1}{2}\{d(x_1, Tx_2) + d(x_2, Tx_1)\}\})) + L\xi(d(x_2, Tx_1)) \\
&\leq \psi(\xi(\max\{d(x_1, x_2), \frac{d(x_1, x_2)d(x_2, Tx_2)}{d(x_1, x_2)}, \frac{1}{2}\{d(x_1, Tx_2) + d(x_2, x_2)\}\})) + Ld(x_2, x_2) \\
&\leq \psi(\xi(\max\{d(x_1, x_2), d(x_2, Tx_2), \frac{1}{2}\{d(x_1, x_2) + d(x_2, Tx_2)\}\})) \\
&\leq \psi(\xi(\max\{d(x_1, x_2), d(x_2, Tx_2)\}))
\end{aligned}
$$

If $\max\{d(x_1, x_2), d(x_2, Tx_2)\} = d(x_2, Tx_2)$,then we have
$0 < \xi(d(x_2, Tx_2)) \leq \psi(\xi(d(x_2, Tx_2))) < \xi(d(x_2, Tx_2))$ which is a contradiction.
Thus,$\max\{d(x_1, x_2), d(x_2, Tx_2)\} = d(x_1, x_2)$,and hence we have
$\xi(d(x_2, Tx_2)) \leq \psi(\xi(d(x_1, x_2))) < \psi^2(\xi(c))$.
Hence,there exists $x_3 \in Tx_2$ such that $\xi(d(x_2, x_3)) < \psi^2(\xi(c))$
Since,T is a α-admissible,from $x_2 \in Tx_1$ and $\alpha(x_1, x_2) \geq 1$,
we have $\alpha(x_2, x_3) \geq 1$.
By induction,we obtain a sequence $\{x_n\} \subset X$ such that,for all $n \in \mathbb{N} \cup \{0\}$,
$\alpha(x_n, x_{n+1}) \geq 1$, $x_{n+1} \in Tx_n$,$x_n \neq x_{n+1}$,$\xi(d(x_n, x_{n+1})) < \psi^n \xi(c)$.
Let $\epsilon > 0$ be given.
Since $\sum_{n=0}^{\infty} \psi^n(\xi(c)) < \infty$,there exists $N \in \mathbb{N}$ such that

$$\sum_{n \geq N} \psi^n(\xi(c)) < \xi(\epsilon) \tag{6}$$

For all $m > n \geq N$,we have

$$
\begin{aligned}
\xi(d(x_n, x_m)) &\leq \sum_{k=n}^{m-1} \psi^k(\xi(c)) \\
&< \sum_{n \geq N} \psi^n(\xi(c)) \\
&< \xi(\epsilon)
\end{aligned} \tag{7}
$$

which implies $d(x_n, x_m) < \epsilon$ for all $m > n > N$.Hence,$\{x_n\}$ is a Cauchy sequence in X.
It follows from the completeness of X that there exists

$$x_* = \lim_{n \to \infty} x_n \in X.$$

Suppose that T is continuous.
We have

$$d(x_*, Tx_*) \leq d(x_*, x_{n+1}) + d(x_{n+1}, Tx_*)$$
$$\leq d(x_*, x_{n+1}) + H(x_n, Tx_*)$$

By letting $n \to \infty$ in the above inequality,we obtain $d(x_*, Tx_*) = 0$,and so $x_* \in Tx_*$.
Assume that f_T is lower semicontinuous.
Then,$f_T(x_*) \leq \lim_{n\to\infty} f_T(x_n)$.
Hence,

$$d(x_*, Tx_*) \leq \lim_{n\to\infty} d(x_n, Tx_n)$$
$$\leq \lim_{n\to\infty} d(x_n, x_{n+1}) = 0..$$

Thus,$x_* \in Tx_*$ □

Corollary 2.2. *Let (X, d) be a comxplete metric space,and let $\alpha \colon X \times X \to [0, \infty)$ be a function.suppose that $T \colon X \to CL(X)$is an α-admissible mappings.*
Assume that for all $x, y \in X$,
$\xi(\alpha(x, y)H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx))$,
where $L \geq 0$,$\xi \in \Xi$,

$$M(x, y) = \max\{d(x, y), \frac{d(x, Tx)d(y, Ty)}{d(x, y)}, \frac{1}{2}\{d(x, Ty) + d(y, Tx)\}\} \quad (8)$$

and $\psi \in \Psi$is strictly increasing .
Also,suppose that conditions(1) and (2) of Theorem 2.1 are satisfied:
Then T has a fixed point in X.

From Theorem 2.1 we obtain the following result.

Corollary 2.3. *Let (X, d) be a comxplete metric space,and let $\alpha \colon X \times X \to [0, \infty)$ be a function.suppose that $T \colon X \to CL(X)$is an α_*-admissible mappings.*
Assume that for all $x, y \in X$,$\alpha(x, y) \geq 1$ implies
$\xi(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx))$,
where $L \geq 0$,$\xi \in \Xi$,

$$M(x, y) = \max\{d(x, y), \frac{d(x, Tx)d(y, Ty)}{d(x, y)}, \frac{1}{2}\{d(x, Ty) + d(y, Tx)\}\} \quad (9)$$

and $\psi \in \Psi$is strictly increasing .
Also,suppose that conditions(1) and (2) of Theorem 2.1 are satisfied:
Then T has a fixed point in X.

Corollary 2.4. *Let (X,d) be a comxplete metric space,and let $\alpha\colon X \times X \to [0,\infty)$ be a function.suppose that $T\colon X \to CL(X)$is an α_*-admissible mappings.*
Assume that for all $x,y \in X$,
$\xi(\alpha(x,y)H(Tx,Ty)) \leq \psi(\xi(M(x,y))) + L\xi(d(y,Tx))$,
where $L \geq 0$,$\xi \in \Xi$,

$$M(x,y) = \max\{d(x,y), \frac{d(x,Tx)d(y,Ty)}{d(x,y)}, \frac{1}{2}\{d(x,Ty)+d(y,Tx)\}\} \quad (10)$$

and $\psi \in \Psi$is strictly increasing .
Also,suppose that conditions(1) and (2) of Theorem 2.1 are satisfied:
Then T has a fixed point in X.

Theorem 2.5. *Let (X,d) be a complete metric space,and let $\alpha\colon X \times X \to [0,\infty)$ be a function.Suppose that a multivalued mappings $T\colon X \to CL(X)$ is α-admissible.*
Assume that,for all $x,y \in X$, $\alpha(x,y) \geq 1$implies
$\xi(H(Tx,Ty)) \leq \psi(\xi(M(x,y))) + L\xi(d(y,Tx))$,
where $L \geq 0$,$\xi \in \Xi$,

$$M(x,y) = \max\{d(x,y), \frac{d(x,Tx)d(y,Ty)}{d(x,y)}, \frac{1}{2}\{d(x,Ty)+d(y,Tx)\}\}$$

and $\psi \in \Psi$is strictly increasing and upper semicontinuous function.Also suppose that the following are satisfied :
(1) there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0,x_1) \geq 1$;
(2)for a sequence $\{x_n\}$ in X with $\alpha(x_n,x_{n+1}) \geq 1$ for all $n \in \mathbb{N} \cup \{0\}$ and a cluster point x of $\{x_n\}$,there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that,for all $k \in \mathbb{N} \cup \{0\}$,
$\alpha(x_{n(k)},x) \geq 1$.
Then T has a fixed point in X.

Proof. Following the proof of Theorem 2.1,we obtain a sequence $\{x_n\} \subset X$ with $\lim_{n\to\infty} x_n = x_* \in X$ such that,for all $n \in \mathbb{N} \cup \{0\}$,
$x_{n+1} \in Tx_n$,$x_n \neq x_{n+1}$,$\alpha(x_n,x_{n+1}) \geq 1$.
From(2) there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k),x_*}) \geq 1$.

Thus,we have

$$\xi(d(x_{n(k)+1}, Tx_*)) = \xi(H(Tx_{n(k),Tx_*})) \quad (11)$$
$$\leq \psi(\xi(M(x_{n(k),x_*}))) + L\xi(d(x_*, x_{n(k)+1})), \quad (12)$$

where

$$M(x_{n(k)}, Tx_*) = \max\{d(x_{n(k)}, x_*), \frac{d(x_{n(k)}, x_{n(k)+1})d(x_*, Tx_*)}{d(x_{n(k)}, x_*)}, \frac{1}{2}\{d(x_{n(k)}, Tx_*) + d(x_*, x_{n(k)+1})\}\}$$

We have

$$\lim_{k\to\infty} M(x_{n(k)}, Tx_*) = d(x_*, Tx_*),$$

and so

$$\lim_{k\to\infty} \xi(M(x_{n(k)}, Tx_*)) = \xi(d(x_*, Tx_*)).$$

Suppose that $d(x_*, Tx_*) \neq 0$.
Since ψ is upper semicontinuous,

$$\lim_{k\to\infty} \psi\xi(M(x_{n(k)}, Tx_*)) = \psi\xi(d(x_*, Tx_*)).$$

Letting $k \to \infty$ in inequality(12) and using continuity of ξ,we obtain

$$\begin{aligned}
0 &< \xi(d(x_*, Tx_*)) \\
&\leq \lim_{k\to\infty} \psi(\xi(M(x_{n(k)}, x_*))) + \lim_{k\to\infty} L\xi(d(x_*, x_{n(k)+1})) \\
&\leq \psi(\xi(d(x_*, Tx_*))) \\
&< \xi(d(x_*, Tx_*))
\end{aligned}$$

which is a contraction .Hence $d(x_*, Tx_*) = 0$,and hence x_* is a fixed point of T. □

The following example shows that upper semicontinuity of ψ cannot be dropped in Theorem 2.5.

Example 2.6. *Let $X = [0, \infty)$,and let $d(x, y) = |x - y|$ for all $x, y \geq 0$ Define a mapping $T : X \to CL(X)$ by*

$$T(x) = \begin{cases} \{\frac{2}{3}, 1\} & (x = 0) \\ \{\frac{5}{6}x\} & (0 < x \leq 1) \\ \{3x\} & (x > 1). \end{cases} \quad (13)$$

Let $\xi(t) = t$ for all $t \geq 0$, and let

$$\psi(t) = \begin{cases} \{\frac{6}{7}t\} & (t \geq 1) \\ \{\frac{5}{6}t\} & (0 \leq t < 1). \end{cases} \tag{14}$$

Then, $\xi \in \Xi$, and $\psi \in \Psi$ and ψ is a strictly increasing function.
Let $\alpha, \eta \colon X \times X \to [0, \infty)$ be defined by

$$\alpha(x, y) = \begin{cases} 6 & (0 \leq x, y \leq 1) \\ 0 & otherwise. \end{cases} \tag{15}$$

Obviously, condition (2) of Theorem 2.5 is satisfied. Condition (1) of Theorem 2.5 is satisfied with $x_0 = \frac{1}{6}$.
We show that (4) is satisfied .
Let $x, y \in X$ be such that $\alpha(x, y) \geq 1$.
Then, $0 \leq x, y \leq 1$.
If $x = y$, then obviously (4) is satisfied .
Let $x \neq y$.
If $x = 0$ and $0 < y \leq 1$, then we obtain

$$\begin{aligned} \xi(H(Tx, Ty)) &= H(\{\frac{2}{3}, 1\}, \frac{5}{6}y) \\ &\leq \frac{1}{6} \leq \psi(d(x, Tx)) \leq \psi(\xi(M(x, y))). \end{aligned}$$

Let $0 < x \leq 1$ and $0 < y \leq 1$.
Then, we have

$$\begin{aligned} \xi(H(Tx, Ty)) &= d(Tx, Ty) = d(\frac{5}{6}x, \frac{5}{6}y) \\ &= \frac{5}{6}|x - y| = \psi(d(x, y)) \\ &\leq \psi(\xi(M(x, y))). \end{aligned}$$

Thus, (4) is satisfied.
We now show that T is αadmissible.
Let $x \in X$ be given, and let $y \in Tx$ be such that $\alpha(x, y) \geq 1$.
Then, $0 \leq x, y \leq 1$.
Obviously, $\alpha(y, z) \geq 1$ for all $z \in Ty$ whenever $0 < y \leq 1$.
If $y = 0$, then $Ty = \{\frac{2}{3}, 1\}$. Hence, for all $z \in Ty$, $\alpha(y, z) \geq 1$.
Hence, T is α-admissible. Thus, all hypotheses of Theorem 2.5 are satisfied. However, T has no fixed points.

Note that ψ is not upper semicontinuous.

Corollary 2.7. *Let (X,d) be a complete metric space,and let $\alpha\colon X\times X\to[0,\infty)$ be a function.Suppose that $T\colon X\to CL(X)$ is an α-admissible mappings.*
Assume that,for all $x,y\in X$, $\xi(H(Tx,Ty))\le\psi(\xi(M(x,y)))+L\xi(d(y,Tx))$, where $L\ge 0$,$\xi\in\Xi$,

$$M(x,y)=\max\{d(x,y),\frac{d(x,Tx)d(y,Ty)}{d(x,y)},\frac{1}{2}\{d(x,Ty)+d(y,Tx)\}\}$$

and $\psi\in\Psi$is strictly increasing and upper semicontinuous function.Also suppose that conditions (1) and(2) of Theorem 2.5 are satisfied.
Then T has a fixed point in X.

Corollary 2.8. *Let (X,d) be a comxplete metric space,and let $\alpha\colon X\times X\to[0,\infty)$ be a function.suppose that $T\colon X\to CL(X)$is an α_*-admissible mappings.*
Assume that for all $x,y\in X$,
$\xi(\alpha(x,y)H(Tx,Ty))\le\psi(\xi(M(x,y)))+L\xi(d(y,Tx))$,
where $L\ge 0$,$\xi\in\Xi$,

$$M(x,y)=\max\{d(x,y),\frac{d(x,Tx)d(y,Ty)}{d(x,y)},\frac{1}{2}\{d(x,Ty)+d(y,Tx)\}\}\quad(16)$$

and $\psi\in\Psi$is strictly increasing .
Also,suppose that conditions(1) *and* (2) *of Theorem 2.5 are satisfied:*
Then T has a fixed point in X.

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